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EMPIRICAL DAMPING CONSTANTS FOR SANDS AND CLAYS

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INTRODUCTION

The dynamic behavior of piling has been of great concern to civil engineers for many years. In 1962, Smith (9) suggested a numerical solution to the pile driving problem. He presented the concept for static loading at the point of a pile such that the ground compresses elastically for a certain distance and then fails plastically with a constant resistance. This concept is illustrated in Fig. 1 by the dotted line OABC. Q in Fig. 1 represents the maximum static elastic ground deformation or quake, and \( R_p \) represents the total ultimate plastic ground resistance to the pile. Under static loading the pile deforms the ground elastically through OA and then plastically through a distance S. The soil then rebounds from B to C leaving a permanent set of S.

Smith (9) developed a mathematical equation which accounts for both static and dynamic soil behavior. Fig. 2 shows the rheological model which simulates the mathematical equation proposed by Smith. The model consists of a spring and friction block in series connected in parallel to a dashpot. If the model were suddenly compressed a certain distance, \( x \), the following equation would describe the soil’s resistance in the elastic region (see Fig. 1):

\[
R_e = K'x + cV
\]

in which \( R_e \) = resisting force; \( K' \) = soil spring constant; \( c \) = a viscous damping constant; \( x \) = elastic deformation of the soil; and \( V \) = the instantaneous velocity of the point of the pile in any time interval. The friction block accounts for the constant soil resistance in the plastic region during static loading and thus does not appear in Eq. 1. In order to include the effect of the pile’s size and shape Smith (9) suggested the following relationship for viscous damping:

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in which $J$ is a viscous damping constant for the soil similar to $c$. As the velocity of deformation approaches zero in Eq. 1, the dynamic resisting force approaches a static value

$$P_{\text{static}} = K' x$$

(3)

Letting $P_{\text{dynamic}}$ equal $R_p$ in Eq. 1 from Smith's mathematical model and substituting Eqs. 2 and 3 into Eq. 1, the peak dynamic resistance of the soil is

$$P_{\text{dynamic}} = P_{\text{static}} (1 + JV)$$

(4)

The concept of the dynamic loading is represented by line OA'/BC of Fig. 1. If $R_p$ in Fig. 1 is the peak static soil resistance, then $R_p JV$ is the dynamic portion of the peak total soil resistance.

This concept for the resistance at the point of the pile takes into account:

(1) Elastic ground deformation; (2) ultimate ground resistance; and (3) viscous damping based on damping constant $J$. Smith assigned a value of $J = 0.15$ for use by investigators until such time that new facts were developed. He pointed out that his mathematical equation could be modified to account for the new facts as they were obtained.

Smith's work was augmented by Samson, Hirsch, and Lowery (8) so that the driving of a pile could be simulated by use of the digital computer. It was their feeling that the resistance to dynamic loading at the point of the pile was not clearly understood and that future study might shed more light on the problem. It is known that the compressive strength of a soil is a function of the time required to reach a failure load. Hampton and Yoder (4) found that in silty clay and clay the unconfined compressive strength showed significant increases with rate of strain for all compactive efforts and all moisture contents tested. Whitman and Healy (10) did an extensive study on shear strength in sands during rapid loading. They developed techniques for applying strains rapidly and measuring resultant stresses and pore pressures, and presented information concerning membrane and inertia effects in triaxial tests. Jones, Lister and Thrower (5) in a related study presented a comprehensive study of the subject of dynamic loading of soils.

APPARATUS, INSTRUMENTATION, AND TEST PROCEDURE

The equipment in this series of tests was necessarily of a special nature. In the dynamic tests it was desired to load the sample over a range of velocities from 0 fps to 12 fps. It was also important that a permanent record of each test be available from which the necessary calculations could be made.

Two separate triaxial devices with load cells in the base were used in this investigation. Fig. 4 shows the cell bases used for both the cohesive and granular materials. The load cells consisted of SR-4 strain gages mounted on the walls of an aluminum tube or pedestal to record the compressional load on imp ct. The cell shown in Fig. 4(a) was developed by Reeves, Coyle, and Hirsch (7) and was used for tests on sands. It has provisions for drainage of the sample at both top and bottom. All sand samples tested in this study were 2.8-in. diam and 6 in. long. The sands were saturated and confined by air pressure in the cell which remained constant during the test. The load cell shown in Fig. 4(b) was developed by Chan, Hirsch, and Coyle (1) and was used for tests on cohesive soils. The cell has no provisions for drainage but it is more sensitive to the smaller loads recorded with cohesive materials. The cohesive specimens used in dynamic tests were of 2.8-in. diam and 3-1/4 in. high. The reason for using shorter cohesive samples is given in the analysis of results of tests on clays.

The loading apparatus was designed and built by Reeves, Coyle, and Hirsch (7) for use in their work on impact loading of sands. A falling weight of 165 lb was sufficient to fail any sample tested, given sufficient height of drop. The drop height could be varied from zero (weight resting on plunger of triaxial device) to 12 in. For tests on dense sands this weight was not large enough to

\[ \text{Chan, Hirsch, and Coyle} \text{ } (1) \text{ investigated in the laboratory the dynamic load deformation and damping properties in sands. Reeves, Coyle, and Hirsch (7) did laboratory research and evaluated the damping constants of sands subjected to impact loads. Using experimental data and Smith's equation, they determined that the damping constant, } J, \text{ was actually a variable for a saturated sand. Coyle and Sulaiman (3) did a study in which they were concerned with the static side friction values encountered in various types of sand. Raba and Coyle (6) investigated frictional damping developed in clays using a model pile in the laboratory. They were able to relate frictional damping to liquidity index for CH materials.} \]
fail the sample when the weight rested on the triaxial cell’s plunger. Therefore, for tests on dense sands, drop heights on the order of 1 in. were used. The frame to stop the falling weight, shown in Fig. 3, could be placed at a height to allow failure of a 6-in. sand sample or could be adjusted to accommodate the shorter 3-1/4-in. samples of cohesive material. A release mechanism allowed the weight to be released instantaneously and to fall freely to impact with the plunger of the triaxial apparatus. The whole frame rested on a steel plate from which was hung 1,400 lb to damp vibrations. The rubber damping pads indicated in Fig. 3 also served this purpose. The falling ram was damped by a 1/4-in. rubber pad to prevent steel-on-steel impact which caused disturbance in the recording system. The velocity of deformation of the sample could be controlled by varying the height of drop. Note that the recorded displacement velocity was higher than the velocity calculated for free falling bodies at the heights shown. The reason for this was that the large ram impacts caused the triaxial plunger to rebound at a slightly greater velocity than the impact velocity. This could be reduced somewhat by putting a thicker rubber pad on the ram.

All static tests were run at a loading rate of 0.05 ipm which is the standard loading rate for compression tests. Measurement of loads for the dynamic and static tests was accomplished using the load cells shown in Fig. 4. Displacement measurements for the dynamic tests were made by means of a linear displacement transducer. As seen in Fig. 3, the displacement transducer was fastened to the triaxial cell and connected to the triaxial plunger, measuring its movement. Displacement measurements for the static tests were made with an Ames dial.

The signals coming from the load cells were channeled into a Carrier Amplifier and a Visicorder Oscillograph, as seen in Fig. 3. The signal from the linear displacement transducer was channeled through a bridge balance unit and then into the Visicorder. The amplifier unit provided a means of amplifying more than one signal simultaneously and the visicorder oscillograph provided a means of representing the signals on photographic paper in a manner yielding the desired information.

A sample visicorder trace is shown in Fig. 5. The left side of the trace shows a calibration curve for the load. In going from A to B a load was placed on the load cell greater than that anticipated on the sample. This load was 1,000 lb which deflected the visicorder point light source by 1.98 in. or 19.8 tenths of an inch. By placing the linear displacement transducer in a device to deflect its shaft 0.1 in., a deflection or attenuation of the point light source to deflect its shaft 0.1 in., a deflection or attenuation of the point light source was achieved. The visicorder channel of 2.25 in. as shown going from C to D was achieved. With the calibration completed, the loads and deflections were again set to zero as seen in lines E and F of the trace. For dynamic tests the visicorder was run at a speed of 80 ips resulting in timing lines on the photographic paper at intervals of 0.01 sec.
Points G and H represent the points of impact between the free falling weight and the sample. At this point, line HI begins to deflect downward from H to I indicating sample deformation and the load trace GJK begins to deflect upward indicating increase in load. Over a very short time interval, the test has been completed. The sample deflection has gone off the paper and the load has returned to zero. It is important to note that the deformation line is straight immediately after contact indicating constant velocity and zero acceleration.

Procedures used in preparation of saturated sand samples were developed by Reeves, Coyle, and Hirsch (7) and used in this study. The cohesive materials were tested in unconfined compression. They were remolded samples prepared by use of a Vac-air extrusion machine. Coyle and Shiffert (2) did considerable work with this machine and have shown that the samples are homogeneous and highly saturated. Raba and Coyle (6) used some of Shiffert's samples in their study. The cohesive samples used in this investigation were prepared in the same manner as those used by Coyle, Shiffert, and Raba (2,6).

RESULTS OF TESTS ON SANDS

For the tests on cohesionless materials, it was desirable to have a reasonably wide range in physical properties. A series of tests were conducted on Ottawa 20-30, Arkansas, and Victoria sands which varied in grain size and grain shape. The Arkansas sand was obtained from a test site at Lock and Dam No. 4 on the Arkansas River. The Victoria sand was obtained from a test site at a highway bridge overpass at Victoria, Texas. The Ottawa sand had grains which were uniform in size and smooth in shape. The Arkansas sand was a fine sand with subangular shaped grains, and the Victoria sand was a very fine sand with angular shaped grains.

The dynamic tests on sands were performed as unconsolidated-undrained tests. The majority of tests were performed at a void ratio of 0.55 and under
a confining pressure of 15 psi. A typical test setup for a dynamic test on sand is shown in Fig. 6. The static tests were performed as consolidated-drained tests, at a void ratio of 0.55 and a confining pressure of 15 psi. The dynamic tests were performed as undrained tests in order to simulate the pore pressure condition at the point of a pile during driving. The static tests were performed as drained tests in order to simulate the drained condition (zero pore water pressure) at the pile point during static loading.

The samples were tested over a range of loading velocities varying from the minimum velocity obtainable to insure sample failure to a maximum velocity of 12 fps. Special care was taken at velocities of sample deformation of 0 fps to 3 fps to determine how the dynamic load varied with velocity in this range. Fig. 7 shows values of peak dynamic load related to velocity of deformation for the three sands tested. The $P_d$ values plotted are the peak values obtained for the static tests which were loaded at the slow rate of 0.05 ipm. The ratio of peak dynamic to peak static load is related to velocity of deformation as shown in Fig. 8.

With velocity of deformation and the ratio of dynamic to static load known, the damping constant, $J$, can be calculated from Eq. 4 by solving for the damping constant

$$J = \frac{1}{2} \left( \frac{P_d}{P_s} - 1 \right)$$

Using Eq. 5 with the experimental laboratory results of this investigation, $J$ values were calculated and results are shown in Fig. 9. As seen in Fig. 9, $J$ is not a constant but varies with velocity of deformation. In order to apply Smith's wave equation analysis (9) to the piling behavior problem, $J$ must be a constant. To obtain a constant $J$, a modification of the original Smith equation was necessary. A reasonably constant value of $J$ was found by raising velocity of deformation to some power less than one. Thus

$$J = \frac{1}{2} \left( \frac{P_d}{P_s} - 1 \right)$$

The results of raising velocity of deformation to a power using Eq. 6 may be seen in Fig. 10 for the tests performed on the three sands.

It was determined from a separate study on each sand that the Ottawa, Arkansas, and Victoria sands have a constant $J$ value when velocities of deformation are raised to $N = 0.21$, $N = 0.27$, and $N = 0.19$ powers, respectively. It was desirable for practical application to represent all three sands by a common value of $N$. The power of $N = 0.20$ was chosen since this gave the least deviation from the optimum power for all three sands. Fig. 10 shows $J$ related to the velocity of deformation for all three sands for the power $N = 0.20$.

In accordance with one of the stated objectives herein, an attempt was made to relate the damping constants obtained to a common sand property. It was
found that the damping constant obtained by using $N = 0.20$ could be related to the effective angle of internal shearing resistance, $\psi'$. This relationship is shown in Fig. 11. The values of $\psi'$ were obtained by conducting drained tests and undrained tests with pore pressure measurements for all three sands at a void ratio of 0.55. There was some question concerning the validity of relating $\psi'$, as determined from a standard laboratory triaxial test, to $J$, as determined from a dynamic test. However, the study made by Whitman and Healy (10) has shown that the difference in dynamic and static angle of internal shearing resistance is less than one degree. As shown in Fig. 11, the correlation between $J$ and $\psi'$ for this study is very good.

A limited number of tests were performed during this study on Ottawa sand in order to determine the change in magnitude of the damping constant, $J$, if the void ratio of the sand was varied. Tests were performed at void ratios of 0.50 and 0.60 with a primary objective of obtaining the relation between peak load and velocity of sample deformation as shown in Fig. 12. The sample at $e = 0.60$ was difficult to prepare because of the extremely loose packing of the grains.

The optimum powers of velocity of deformation to obtain a constant $J$ for the $e = 0.50$ and $e = 0.60$ tests are quite different from the value of $J$ for $N = 0.20$. Fig. 13 shows a considerable deviation in $J$ values which results when velocity of deformation is raised to both the optimum $N$ value and $N = 0.20$. The major deviations in $J$ values are seen to occur at the loosest void ratio of $e = 0.60$. It is felt that if a pile were driven in sands with a void ratio as loose as $e = 0.60$, the sands would consolidate to a denser void ratio during driving. Thus, considering the denser void ratios in Fig. 13, the average $J$ values shown by representing velocity of deformation to the $N = 0.20$ power are acceptable.

The significance of these relationships is that in clean sands, if the void ratio of a particular material or the effective angle of internal shearing resistance is known, an approximation of a $J$ value can be obtained.

**RESULTS OF TESTS ON CLAYS**

For the tests on cohesive materials, it was again desirable to vary the physical properties. A series of tests was conducted on four clay materials. Three of the soils were classified as CH by the Unified Soil Classification System and the fourth soil was classified as CL. One of the CH soils was an organic clay which had a liquid limit of 53 and was tested at a moisture content of 36% (Test Soil—OR 36). The other CH soils tested were local soils named Easterwood clay and Vettes clay. The Easterwood clay had a liquid limit of 94 and was tested at moisture contents of 50% and 60% (Test Soils—EA 50 and EA 60). The Vettes clay had a liquid limit of 80 and was tested at moisture contents of 46%, 50%, and 55% (Test Soils—VE 46, VE 50, and VE 55). The CL soil tested was a hall pit clay with a liquid limit of 48 and was tested at a moisture content of 35% (Test Soil—CE 35).

The dynamic tests on the clays were performed as unconsolidated-undrained tests with no confining pressure. The static tests were performed in the same manner as a standard unconfined compression test. There was some question concerning the effect of confinement on the clay soils. A preliminary study was made using the organic material at 36% moisture content (OR 36) in order...
to evaluate the effect of confinement. Unconsolidated-undrained tests were conducted at two confining pressures (15 psi and 30 psi). The results of these dynamic tests are shown in Fig. 14. The confinement caused only a slight increase in the peak loads. Since the effect of confinement was minimal, it was decided that the test program should involve only unconfined tests.

The cohesive materials were tested over a range of loading velocities of from 0 fps to 12 fps. Data were reduced from the visicorder trace using the same procedure that was used for the sands. Fig. 15 shows the values of peak dynamic load related to velocity of deformation for the clays tested. As in the case of the saturated sands, the peak dynamic loads in the clays increased rapidly at low velocities and then leveled off to an essentially straight line with a slight slope. Note that the slopes are nearly parallel, and that the peak loads increase as the moisture content decreases for a given clay soil. Fig. 16 shows the values of the ratio of dynamic to static loads related to velocity of deformation for the clays tested.

Using the results from the clay tests, it was possible to compute the damping constant, $J$, with Smith's equation (Eq. 5). A typical curve showing Smith's $J$ related to velocity of deformation is shown in Fig. 17 for the EA 50 material.
The values of $J$ and the optimum power to which velocity of deformation must be raised in order to obtain a constant $J$. Again, for purposes of practical application the velocity of deformation was raised to one common power for all clays. This common power of velocity of deformation was $N = 0.18$ for the materials tested. This power is not an average value but rather a number arrived at by inspecting the relative change in the data brought about by a change in power of velocity of deformation. Fig. 18 shows, for the materials tested, the $J$ value related to velocity of deformation raised to the 0.18 power. It was possible, with the test results obtained, to correlate the damping constants with several common clay properties. For a given clay soil, the $J$ values could be related to moisture content. Fig. 19 shows the $J$ values obtained when velocity of deformation was raised to the $N = 0.18$ power related to moisture content for the Vetters clay. An essentially linear relationship exists and similar relationships were obtained for the other clay soils.

In addition to moisture content correlation, it was possible to relate the $J$ values for the CH materials to liquidity index. Liquidity index is defined as:

$$LI = \frac{\text{Natural Moisture Content} - \text{Plastic Limit}}{\text{Plasticity Index}}$$

Fig. 20 shows the $J$ values related to liquidity index. The liquidity index was considered an important parameter in this study because it includes the Atterberg Limits as well as the moisture content of the clay. The data shown in Fig. 20 include some test results from a preliminary test program conducted in the fall of 1967. Generally, the results of tests performed in this study (spring 1968) lie above the earlier test results due to thixotropic hardening of the clay samples. All of the data are shown as lying within a band, and the dotted lines on Fig. 20 show that maximum deviation was about 12%. The hall pit clay does not fall into this band, and since this material was a CL it appears that the band is only valid for CH materials. Perhaps if more tests had been conducted on CL materials, a different band would be established for them.

The significance of these relationships for clays is that if properties such as moisture content and liquidity index are known, then an approximate value for the damping constant, $J$, can be established.

**CONCLUSIONS**

The following conclusions can be made concerning this study:

1. When the experimental laboratory data from this study were used with Smith's equation (Eqs. 4 and 5), the damping constant, $J$, varied with velocity of deformation for all materials tested. (See Fig. 9 for sands and Fig. 17 for clays.)
2. If Smith's equation was modified so that velocity was raised to some power, $N$, less than 1.0 (Eq. 6), then a reasonably constant value for $J$ was obtained for all values of velocity from 0 fps to 12 fps. (See Fig. 10 for sands and Fig. 18 for clays.)
3. An acceptable constant $J$ value for saturated sands may be obtained by raising velocity of deformation to the power of $N = 0.20$ (See Fig. 10).
4. An acceptable constant $J$ value for clay may be obtained by raising the velocity of deformation to the power of $N = 0.18$ (see Fig. 18).
5. An approximate J value for saturated sand may be obtained if the effective angle of internal shearing resistance is known (see Fig. 11).

6. An approximate J value for clay (Classification—CH) may be obtained if the liquidity index is known (see Fig. 20).

It should be remembered that all data collected herein were the result of laboratory tests conducted on prepared samples. The failure mechanism occurring in the soil sample tested in the laboratory may not be the same as the failure mechanism occurring at the tip of a full scale pile in the field. However, results from current studies being conducted at Texas A&M University in field soils indicate that the damping constants obtained in this study are useable with field piles, especially in saturated sands.

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APPENDIX I.—REFERENCES


APPENDIX II.—NOTATION

The following symbols are used in this paper:

\[ \begin{align*}
CE 35 &= \text{Hall Pit clay at an approximate moisture content of 35\%;} \\
EA 50 &= \text{Eastwood clay at an approximate moisture content of 50\%;} \\
EA 60 &= \text{Eastwood clay at an approximate moisture content of 60\%;} \\
e &= \text{void ratio;} \\
J &= \text{viscous damping constant for soil, in seconds per foot;} \\
k &= \text{spring constant for soil mass segment, in pounds per inch;} \\
N &= \text{power to which velocity of sample deformation is raised;} \\
o &= \text{elastic deformation of the soil, in inches;} \\
\eta &= \text{permanent set of the soil, in inches;} \\
\rho &= \text{resisting force of the soil in the elastic region, in pounds;} \\
p_{\text{dynamic}} &= \text{dynamic strength of soil, in pounds;} \\
p_{\text{static}} &= \text{static strength of soil, in pounds;} \\
p &= \text{total ultimate plastic ground resistance, in pounds;} \\
S &= \text{permanent set of the soil, in inches;} \\
V &= \text{velocity of deformation of the soil, in feet per second;} \\
VE 46 &= \text{Vetters clay at an approximate moisture content of 46\%;} \\
VE 50 &= \text{Vetters clay at an approximate moisture content of 50\%;} \\
VE 55 &= \text{Vetters clay at an approximate moisture content of 55\%;} \\
6^\circ &= \text{effective angle of internal shearing resistance, in degrees.}
\end{align*} \]