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We Shall Achieve

Forward and Inverse Solution of the Wave Equation for Piling Using Axisymmetric Finite Elements

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Objective

• Develop an improved method for the static and dynamic analysis of driven piles for both forward and inverse solutions
  
  • Forward methods: the hammer is modeled and the pile response and capacity for a certain blow count is estimated
  
  • Inverse methods: the force-time and velocity- or displacement-time history from driving data is used to estimate the pile capacity
Driven Piles

• Oldest type of deep foundation in use
• Still common in many structures, especially for transportation applications
Wave Propagation in Piles

\[ \frac{\partial^2}{\partial t^2} u(x, t) = c_a^2 \frac{\partial^2}{\partial x^2} u(x, t) \]

- First became an issue with concrete piles
- First identified and studied in 1930's
- Smith (1960) developed the first viable numerical analysis of wave propagation in piles
Inverse Methods for Piles

- Geotechnical uncertainties made use of driving results important
- Dynamic formulae, based on rigid body Newtonian impact mechanics, were popular but inadequate to the task of predicting SRD (soil resistance to driving)
- First research on topic was in late 1950's, but Rausche (1970) developed the first viable technique to use wave propagation theory to predict SRD from field data
- Developed methods such as the Case Method and CAPWAP
CAPWAP and Pile Driving Analyzer

1. Measure $F_m, a_m$
2. Compute $F_c = F_d(a_m, R_s, R_l)$
3. Compare $F_m \sim F_c$
4. Correct $R_s, R_l$
5. Iterate (go to 2)
We Shall Achieve

Weaknesses of Current State of the Art

- Limitations of one-dimensional Smith visco-elastic-plastic model
- Lack of clear relationship between basic soil properties and properties used in wave equation analysis
- Uniqueness issues regarding soil model with first and second time derivatives
- Uncertainties in relationship between pile axial capacity and SRD
Elements of Proposed Solution: Program STADYN

- Two/Three-Dimensional Axisymmetric Analysis of Hammer, Pile and Soil System
- Mohr-Coulomb Elasto-Perfectly Plastic Solution
- Galerkin Finite Element Solution, Static and Dynamic Problems with Same Model
- Aggregation of Soil Properties for Simplified Input and Inverse Solution
- Polytope Optimization Technique for Determination of SRD From Field Data
Two/Three-Dimensional Axisymmetric Analysis of Hammer, Pile and Soil System
Two/Three-Dimensional Axisymmetric Analysis of Hammer, Pile and Soil System
Mohr-Coulomb Elasto-Perfectly Plastic Solution
Galerkin Finite Element Solution

Corner Nodes: \( \frac{1}{4} (1 - \xi_i)(1 - \eta_i) \), \( i = 1, 2, 3, 4 \)
Aggregation of Soil Properties for Simplified Input and Inverse Solution

- Modulus of Elasticity $E$
- Poisson’s Ratio $\nu$
- Dry Density of Soil $\rho$
- Cohesion $c$
- Yield Strength of Soil $\sigma_{\text{yield}}$. This is in reality $\sigma_{\text{yield}} = q_c = 2c$ (67)
- Internal Friction Angle of Soil $\phi$
- Dilitancy Angle of Soil $\psi$. This was discussed earlier; it is either set to zero or is a function of $\phi$.
- Acoustic Speed of Soil or Other Material $c_a$, which is determined by $c_a = \sqrt{\frac{E}{\rho}}$ (68)
- Specific Gravity of the soil particles, $G_s$. 

(Density or Consistency)

$\eta$

$\xi$ (Degree of Cohesion)
Polytope Optimization Technique for Determination of SRD From Field Data
Test Cases

- Fixed Base Study
- Dilitancy and Element Squeeze
- Static Load Test Interpretation
- Modeled Hammer Runs, Cushioned and Cushionless
- Bearing Graph Study
- Comparison with Actual Static Load Test and Driving Record
- Comparison with GRLWEAP
- Inverse Study Comparison with CAPWAP
Inverse Study Using Optimization Technique

- Job Site: Test Pile in Kenner, Louisiana
- Composite Pile: 172 mm O.D. Steel Pipe Pile, 13.72 meters long, with 305 mm 10.67 m long timber “stinger”
- Hammer: Vulcan SC-9
- Driving into soft to medium fat clays, saturated virtually to soil surface
- Cases for 1, 2, 4, 8 and full (24) soil layers
Pile Head Test Data Input
FEA Model, Static and Dynamic
Displacement-Time Results
Stress-Time Results
Optimization Track (Annealed)
Velocity Time Track (Four Layer Annealed Case)
Soil Aggregate Properties Result (Four-Layer)
SRD Comparison

![Graph showing the comparison between Standard Polytope and Annealed Polytope for different numbers of shaft layers, with decreasing load capacity as the number of layers increases.](image)
Conclusions

- Basic validity of the model was established
- Addition of “convenience features” (layering, etc.) was very helpful
- Aggregation technique was helpful with inverse method but needs further calibration with actual soil properties
- Other specific issues to driven piles need to be resolved